**Effect of adding mass to pendulum on period time**

**Rationale**

A pendulum is a body suspended from a fixed point so that it can swing back and forth under the influence of gravity (The Editors of Encyclopaedia Britannica, 2021). Pendulums are used in many everyday inventions, some obvious, such as the pendulum clock and some less so, such as swing sets or seismometers. Pendulums function by utilising the fact that acceleration due to gravity is always downwards and is towards its equilibrium point. This downwards velocity produced by gravity is then converted into horizontal velocity because of the rotation of the object. Assuming no external factors such as friction or air resistance, the relationship between the period time, pendulum length and acceleration due to gravity of a pendulum can be found in the following formula.In this formula it can be seen that mass is absent as, acceleration due to gravity is constant for all objects regardless of their mass. This is phenomenon is also found in the experiment of dropping a bowling ball and feather in a vacuum. The initial angle is also missing because although the pendulum experiences higher displacement with higher angles, it travels faster as it accelerates for a longer time. These two effects cancel each other out, causing the period of the pendulum to remain the same regardless of the initial angle. This formula however, assumes small angle approximation. Small angle Approximation states that (Russell, 2011). This is assumed when deriving the formula for period above, as the true formula is a second order differential equation, which is out of the scope of this report. This evidence has resulted in the following research question.

*Do changes to mass result in a constant period of a pendulum, while pendulum length and initial angle remains constant?*

**Method**

Materials:

* Retort stand
* String
* Slotted masses and stand
* Protractor
* Stopwatch
* Counterweight (books)
* Ruler

1. Set up retort stand with books at base of stand and attach string to retort and slotted mass stand. Attach protractor to stand to measure initial angle.
2. Measure 30 cm from where the string is attached to the stand to the bottom of the slotted mass stand.
3. Add weight to make 100 grams. Hold the pendulum to the initial angle of 45 degrees and release while simultaneously starting a stopwatch.
4. Watch the pendulum closely and count five periods before stopping the stopwatch.
5. Record the time and repeat for 5 trials and masses from 100 to 500 grams with 100-gram intervals.

**Method Modifications**

In the original method, a ball is dropped at various heights and the time it takes to impact the ground is measured with a stopwatch. In the modified experiment used for this report, instead, a pendulum is released at a constant angle and the time it takes for five periods to occur is measured. This is repeated with increasing amounts of mass on the pendulum to find a relationship between mass and the period. Five trials of five periods were used instead of three trials in the previous experiment. This is because it significantly increases the amount of sample of data to give a more precise average time. The pendulum was used because it is easier to drop consistently and time, as the person using the stopwatch can more accurately anticipate when to start and stop. An angle of 45 degrees was chosen to be the initial angle, as it was easy to find on a protractor and seemed reasonable.

**Safety**

|  |  |  |
| --- | --- | --- |
| Hazard | Injury | Control + Management |
| Falling Masses | Bruised foot | Keeping the masses together and avoiding knocking them off the table |
| Pendulum Detaching from string | Blunt Trauma | Verifying the knot used by pulling on it before use |
| Retort stand falling over | Blunt Trauma | Putting heavy books on the base of the stand to stop it from wobbling and wearing closed in shoes |

Due to no chemicals, or environmental factors, no ethical considerations were necessary.

**Results**

Table 1: Raw results of 5 periods and average

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Mass (Kg) | Time (s)  Trial 1 | Time (s)  Trial 2 | Time (s)  Trial 3 | Time (s)  Trial 4 | Time (s)  Trial 5 | Time (s)  Average |
| 0.1 | 5.22 | 5.4 | 5.25 | 5.19 | 5.41 | 5.294 |
| 0.2 | 5.16 | 5.22 | 5.34 | 5.16 | 5.31 | 5.238 |
| 0.3 | 5.13 | 5.18 | 5.34 | 5.13 | 5.28 | 5.212 |
| 0.4 | 5.22 | 5.06 | 5.47 | 5.09 | 5.12 | 5.192 |
| 0.5 | 4.94 | 5.28 | 5.16 | 5.21 | 5.12 | 5.142 |

Table 2: Average period in each trial in seconds and std dev.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Trial 1 Average Period (s) | Trial 2 Average Period (s) | Trial 3 Average Period (s) | Trial 4 Average Period (s) | Trial 5 Average Period (s) | Average Period (s) | Standard Deviation |
| 1.044 | 1.08 | 1.05 | 1.038 | 1.082 | 1.0588 | 0.02 |
| 1.032 | 1.044 | 1.068 | 1.032 | 1.062 | 1.0476 | 0.01 |
| 1.026 | 1.036 | 1.068 | 1.026 | 1.056 | 1.0424 | 0.01 |
| 1.044 | 1.012 | 1.094 | 1.018 | 1.024 | 1.0384 | 0.03 |
| 0.988 | 1.056 | 1.032 | 1.042 | 1.024 | 1.0284 | 0.02 |

Calculating Average Period in trial 1 mass 0.1 kg

Calculating Average Period across trials for 0.1 kg

Calculating Standard Deviation for 0.1 Kg

Calculating Theoretical Period assuming small angle approximation

Figure 1: Average period Time graphed against pendulum mass

**Analysis & Interpretation**

As seen in figure one, there is a very slight decrease in period time as more mass is added to the pendulum. The amount of uncertainty also trended upwards simultaneously. The uncertainty ranged from 1.6 to 3.2 percent, which is considered precise. The coefficient of determination of the linear trend line is also very high at 0.96 which also suggests a precise measurement.

Because all factors affecting period time are known, a theoretical period was found to be 1.099 with small angle approximation and 1.143 using an online mathematical engine which utilises the initial angle of 45 degrees (Wolfram Alpha, 2023). These values are greater than all recorded times during the experiment. This implies there is some systematic error in the method of the experiment causing this variation from the theoretical value. This error is related to the mass rather than measurements in time or length, as when the mass is increased it becomes further from the theoretical value.

**Evaluation**

The decrease in period time whenever the mass was increased was partly due to the decrease in distance from the top of the retort stand to the centre of mass of the hanging body. This was caused by the design of the stand used to hold the masses. By stacking the masses one on top of the other, the centre of mass of the stand raised over the course of the experiment. As seen in the formula for the period of the pendulum, a decrease in the length of the pendulum causes a decrease in period time. This is clearly seen in the results of the experiment. The other issue of increasing uncertainty as mass increased is due to of the person's timing tiring throughout the course of the experiment. This means that measurements for the higher masses are less reliable, however do appear to be accurate. Because the initial angle of the pendulum was consistent at 45 degrees, variations of this did not have a significant impact. Another force impacting the pendulum during the experiment was that of air resistance. Theoretically, air resistance should slow the pendulum and therefore increase the period. Since the experimental results show a value less than that of the theoretical, this suggests that this force did not significantly change the final result

**Improvements and extensions**

To fix some of the previously documented limitations, further changes to the method should be implemented. To fix the issue of varying centre of mass between increasing mass, the length of the string should be reduced to thirty centimetres minus the distance from the bottom of the pendulum to the centre of mass. This should be done while changing the number of masses. This will ensure that the true length of the pendulum is consistent throughout the experiment. To improve the uncertainty from the data rather than using a stopwatch to measure the period time, a video camera should be used with conjunction with further technology to find precisely the time between when the pendulum is dropped to when it has performed five periods. To decrease error caused by small angle approximation in the motion of a pendulum formula and air resistance a small initial angle should be used of 10 degrees.

The research question could be expanded to include more factors which theoretically do not affect the period time of a pendulum such the pendulum rigidness. It could also be extended to create an approximation for gravity and compare that to the theoretical value of 9.81 rather than comparing the theoretical values of the pendulum period.

**Conclusion**

Changes to mass do not affect period of a pendulum while pendulum length and acceleration due to gravity remain constant. This was seen in the results of the experiment which clearly support this claim as the variation in the results can be explained by the change in centre of mass and not the mass of the pendulum itself.

Word Count: 1541

**References**

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